Standard complex stuff

$z = re^{i\theta}$; $\boldsymbol{\theta \in (-\pi, \pi]}$	$z^n = r^n(\cos n\theta + i\sin n\theta)$	$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right)$; $\{\boldsymbol{k \in \mathbb{Z}: k \in [0, n-1]}\}$
$e^z = e^{x+yi} = e^x(\cos y + i\sin y)$ $e^{iz} = \cos z + i\sin z$		$\ln z = \ln r + i(\theta + 2k\pi)$; $\{\boldsymbol{k \in \mathbb{Z}: (\theta + 2k\pi) \in (-\pi, \pi]}\}$
$\sin z = \frac{1}{2i}\left(e^{iz} - e^{-iz}\right)$; $\cos z = \frac{1}{2}\left(e^{iz} + e^{-iz}\right)$		$\sinh z = \frac{1}{2}(e^z - e^{-z})$; $\cosh z = \frac{1}{2}(e^z + e^{-z})$
$\sinh z = -i\sin iz$; $\cosh z = \cos iz$		$\overline{z_1} + \overline{z_2} = \overline{z_1 + z_2}$; $\overline{z_1} \times \overline{z_2} = \overline{z_1 \times z_2}$

Calculus stuff

Analytic: $u_x = v_y$; $u_y = -v_x$	**Entire:** Analytic in $\mathbb{C}$
Harmonic: $u_{xx} + u_{yy} = 0$; $v_{xx} + v_{yy} = 0$	**Exact differential:** $df(x,y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$
Given $\boldsymbol{u(x,y)}$ to find $\boldsymbol{f(z) = u(x,y) + iv(x,y)}$ ① Check harmonic: $u_{xx} + u_{yy} = 0$ ② Let $f(z) = \int f'(z)\,dz = \int (u_x - iu_y)\,dz$	$\int_C f(z)\,dz = \int_a^b f\big(z(t)\big)\,z'(t)\,dt : t \in [a,b]$
$f(z)$ is analytic, $C: z_0 \to z_1$: $\int_C f(z)dz = \int_{z_0}^{z_1} f(z)dz$	$f(z)$ is analytic, $\boldsymbol{C}$ **encloses** $\boldsymbol{z_0}$, anti-clockwise: $\oint_C \frac{f(z)}{(z-z_0)^n}dz = \frac{2\pi i}{(n-1)!}f^{(n-1)}(z_0)$

Sequence & Series stuff

$\lim_{n\to\infty}\left\|\frac{z_{n+1}}{z_n}\right\| \begin{cases} < 1 & \text{convergent} \\ > 1 & \text{divergent} \\ = 1 & \text{inconclusive} \end{cases}$		$s = \sum_{n=0}^{\infty} a_n(z-z_0)^n$; $R = \lim_{n\to\infty}\left\|\frac{a_n}{a_{n+1}}\right\|$	
Taylor series: $f(z) = \sum_{n\geq 0}\frac{1}{n!}f^{(n)}(z_0)\,(z-z_0)^n$		$\ln(1-z) = -\sum_{n\geq 1}\frac{1}{n}z^n$	
$e^z = \sum_{n\geq 0}\frac{1}{n!}z^n$	$\sin z = \sum_{n\geq 0}\frac{(-1)^n}{(2n+1)!}z^{2n+1}$ $\cos z = \sum_{n\geq 0}\frac{(-1)^n}{(2n)!}z^{2n}$	$\sinh z = \sum_{n\geq 0}\frac{1}{(2n+1)!}z^{2n+1}$ $\cosh z = \sum_{n\geq 0}\frac{1}{(2n)!}z^{2n}$	$\frac{1}{z+a} = \frac{1}{a}\sum_{n\geq 0}\left(\frac{-1}{a}z\right)^n$

Ordinary differential equation stuff

Given $\boldsymbol{y' + A(x)y = B(x)}$ $y = \frac{1}{\alpha(x)}\left(\int \alpha(x)\,B(x)\,dx + C\right)$; $\alpha(x) = e^{\int A(x)\,dx}$	**Given $\boldsymbol{P(x,y)\,dx + Q(x,y)\,dy = 0}$** ① Check exact: $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ ② $\int_0^x P(u,y)\,du + \int_0^y Q(0,v)\,dv = C$
Given $\boldsymbol{g(x,y,y',y'') = f(x)}$ ① Solve y_h for $f(x) = 0$ ② Let y_p be a linear combination of terms in the closed set of the terms in $f(x)$, compute y_p', y_p'' ③ $y = y_h + y_p$	**Given $\boldsymbol{u_x, u_y}$ to find $\boldsymbol{u(x,y)}$** ① $u = \int u_x\,dx + h(y)$ ② $u_y = \frac{\partial u}{\partial y} = \frac{\partial(\int u_x\,dx)}{\partial y} + h'(y)$ ③ $u = \int u_x\,dx + \int h'(y)\,dy$
Given $\boldsymbol{ay'' + by' + cy = 0}$ $y_h = \begin{cases} C_1e^{r_1x} + C_2e^{r_2x} & r_1 \neq r_2 \\ e^{rx}(C_1x + C_2) & r_1 = r_2 \\ e^{\alpha x}(C_1\sin\beta x + C_2\cos\beta x) & r = \alpha \pm \beta i \end{cases}$; $\boldsymbol{ar^2 + br + c = 0}$	**Solutions linear independence:** $W = \begin{vmatrix} y_1^{(0)} & \cdots & y_n^{(0)} \\ \vdots & \ddots & \vdots \\ y_1^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix} \neq 0$
Given $\boldsymbol{\sum_{i=0}^n a_iy^{(i)} = 0}$ $y_h = \sum_{i=1}^k\left(e^{r_ix}\sum_{j=0}^{m_i-1}C_{ij}x^j\right)$; $\boldsymbol{\sum_{i=0}^n a_ir^i = \prod_{i=1}^k (x - r_i)^{m_i} = 0}$	**Change of variables:** $x = e^u$ $y = v$ $y' = e^{-u}v'$ $y'' = e^{-2u}v'' - e^{-2u}v'$
Given $\boldsymbol{ax^2y'' + bxy' + cy = 0}$ $y_h = \begin{cases} C_1x^{r_1} + C_2x^{r_2} & r_1 \neq r_2 \\ x^r(C_1\ln x + C_2) & r_1 = r_2 \\ x^\alpha[C_1\sin(\beta\ln x) + C_2\cos(\beta\ln x)] & r = \alpha \pm \beta i \end{cases}$; $\boldsymbol{ar^2 + (b-a)r + c = 0}$	

Partial differential equation stuff

Partial integration: Let all constants $C_n = f_n(y)$ for $\int u(x,y)\,dx$	**Given $au_{xx} = u_t$** $u_i(x,t) = (A\sin\kappa x + B\cos\kappa x)e^{-a\kappa^2 t} + Cx + D$

Given $a^2u_{xx} = u_t$; $u\|_{x=0} = c$; $u\|_{x=L} = d$; $u\|_{t=0} = f(x)$

$$u(x,t) = c + \frac{d-c}{L}x + 2\sum_{n=1}^{\infty} k_n \sin\frac{n\pi}{L}x\, e^{-\left(\frac{n\pi a}{L}\right)^2 t};\ k_n = \frac{1}{L}\int_0^L \left(f(x) - c - \frac{d-c}{L}x\right)\sin\frac{n\pi}{L}x\,dx$$

Given $u_{tt} = c^2u_{xx}$; $u|_{x=0} = u|_{x=L} = 0$; $u|_{t=0} = f(x)$; $u_t|_{t=0} = g(x)$

① $u(x,t) = 2\sum_{n=1}^{\infty}\sin\frac{n\pi}{L}x\left(a_n\cos\frac{n\pi c}{L}t + b_n\sin\frac{n\pi c}{L}t\right)$

② $\begin{cases} a_n = \frac{1}{L}\int_0^L f(x)\sin\frac{n\pi}{L}x\,dx \\ b_n = \frac{1}{cn\pi}\int_0^L g(x)\sin\frac{n\pi}{L}x\,dx \end{cases}$

Given $u_{xx} + u_{yy} = 0$; $u|_{x=0} = u|_{x=a} = u|_{y=0} = 0$; $u|_{y=b} = f(x)$

$$u(x,y) = 2\sum_{n=1}^{\infty}\left(d_n\sin\frac{n\pi}{a}x\sinh\frac{n\pi}{a}y\right);\ d_n = \frac{1}{a\sinh\frac{n\pi b}{a}}\int_0^a f(x)\sin\frac{n\pi}{a}x\,dx$$

Given $P(x,y,u)u_x + Q(x,y,u)u_y = R(x,y,u)$; $u\|_{y=0} = f(x)$ ① $\frac{\partial x}{P(x,y,u)} = \frac{\partial y}{P(x,y,u)} = \frac{\partial u}{R(x,y,u)}$; $\begin{cases} Q\,dx = P\,dy \\ R\,dx = P\,du;\ \ R\,dy = Q\,du \end{cases}$ ② $F(\int Q\,dx - \int P\,dy) = \int R\,dx - \int P\,du = \int R\,dy - \int Q\,du$ ③ Substitute $u(x,0) = f(x)$ to find $F(t)$ ④ Find $u(x,y)$	**Given $eq[u, u_x, u_t]$** ① Let $u(x,t) = X(x)\cdot T(t)$ ② $f(X,X') = g(T,T') = k$ ③ Reduce into ODEs ④ Find $X(x)$, $T(t)$ ⑤ Find $u(x,t)$

Fourier stuff

Given $f(x)$ of period $2r$ ① $f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{r}x + b_n\sin\frac{n\pi}{r}x\right)$ ② $\begin{cases} a_0 = \frac{1}{2r}\int_{-r}^{r} f(x)\,dx \\ a_n = \frac{1}{r}\int_{-r}^{r} f(x)\cos\frac{n\pi}{r}x\,dx \\ b_n = \frac{1}{r}\int_{-r}^{r} f(x)\sin\frac{n\pi}{r}x\,dx \end{cases}$	**Given $f(x)$ for $x \in [0,r]$ to extend _evenly_ to period $2r$** ① $f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{r}x\right)$ ② $\begin{cases} a_0 = \frac{1}{r}\int_0^r f(x)\,dx \\ a_n = \frac{2}{r}\int_0^r f(x)\cos\frac{n\pi}{r}x\,dx \end{cases}$ **Given $f(x)$ for $x \in [0,r]$ to extend _oddly_ to period $2r$** $f(x) = \sum_{n=1}^{\infty}\left(b_n\sin\frac{n\pi}{r}x\right); b_n = \frac{2}{r}\int_0^r f(x)\sin\frac{n\pi}{r}x\,dx$

Calculus identities

$d(\sin x) = \cos x\,dx$	$d(\cos x) = -\sin x\,dx$	$d(\tan x) = \sec^2 x\,dx$
$d(\sec x) = \sec x\tan x\,dx$	$d(\csc x) = -\csc x\cot x\,dx$	$d(\cot x) = -\csc^2 x\,dx$
$\int\sec x\,dx = \ln\|\tan x + \sec x\| + C$	$\int a^x\,dx = \frac{1}{\ln a}a^x + C$	$\int\ln x\,dx = x\ln x - x + C$
$\int\csc x\,dx = -\ln\|\csc x + \cot x\| + C$	$\int\frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C = -\cos^{-1}x + C$	
$\int\tan x\,dx = -\ln\|\cos x\| + C$	$\int\frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x + C = -\csc^{-1}x + C$	
$\int\cot x\,dx = \ln\|\sin x\| + C$	$\int\frac{1}{1+x^2}dx = \tan^{-1}x + C = -\cot^{-1}x + C$	

Trigonometric identities

$\sin^2 x + \cos^2 x = 1$	$\sec^2 x - \tan^2 x = 1$	$\csc^2 x - \cot^2 x = 1$
$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$	$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	
$\sin 2x = 2\sin x\cos x$	$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$	
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$; $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$	$\tan x = \frac{\sin 2x}{1+\cos 2x} = \frac{1-\cos 2x}{\sin 2x} = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$	
$\sin 3x = 3\sin x - 4\sin^3 x$	$\cos 3x = 4\cos^3 x - 3\cos x$	
$\sin x\sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$ $\cos x\cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$ $\sin x\cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$	$\sin x \pm \sin y = 2\sin\frac{x\pm y}{2}\cos\frac{x\mp y}{2}$ $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$ $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$	